

Dynamic, Simulation and Control Design of an Unmanned Hovercraft

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Abstract—A simplified model of the hovercraft is used having three degrees of freedom and the control is considered as having two inputs. This paper addresses the control law problem by reformulating the problem in terms of a direct Lyapunov approach whose derivation is performed using the symbolic manipulation program Maple. The proposed solution for this controller design formulation uses the control law instead of inverse dynamics to determine the coordinate histories for the unspecified axes, and represents a novel approach for the control of the underactuated system such that the control law could stabilize both the actuated and underactuated axes. Simulation of the model is carried out in the MATLAB/Simulink environment, bringing a new effective method to solve the control problem of the hovercraft, which is a difficult system to control because its movement is subjected to nonholonomic constraints.

Keywords—controller, hovercraft, Lyapunov, model, movement, nonholonomic, underactuated.

Resumen—Se presenta un modelo simplificado del aerodeslizador utilizando tres grados de libertad y considerando el control como uno de dos entradas. En este trabajo se aborda el problema de la ley de control reformulando el problema en términos de un enfoque directo de Lyapunov cuya derivación se realiza utilizando el programa de manipulación simbólica Maple. La solución propuesta para esta formulación diseño del controlador utiliza la ley de control en lugar de la dinámica inversa para determinar el historial de coordenadas para los ejes no especificados, y representa un enfoque novedoso para el control del sistema de subactuado de tal manera que la ley de control podría estabilizar tanto al eje actuado como

al subactuado. Simulación del modelo se lleva a cabo en el entorno MATLAB / Simulink, con lo que un nuevo método eficaz para resolver el problema de control del aerodeslizador, que es un sistema difícil de controlar debido a su movimiento se somete a restricciones no holonómicas.

Palabras Claves— controlador, aerodeslizadores, Lyapunov, modelo, movimiento, no holonómica, subactuado.

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1. Introduction

The use of unmanned vehicles (UVs) gains interest due to the various applications that UVs provide as solutions to the many needs of society.

Compared with traditional UVs, the hovercraft provides faster speeds for running on water, ice, and land surfaces. A control law for the asymptotic stabilization of an unmanned hovercraft is accomplished by finding the solution of matching conditions that arise from Lyapunov's second method, analogous to the dissipation of energy, [1]. A simplified model of the hovercraft is used having three degrees of freedom and the control is considered as having two inputs. This controller

design formulation is exact without introducing approximations, and it is proposed to determine the coordinate histories for the unspecified axes, such that the control law could stabilize both the actuated and underactuated axes [2].

A new method will be tested and developed under different conditions, in order to achieve high performance and a stable hovercraft prototype given that the hovercraft is not actuated in the lateral direction. Figure 1 shows the details of the prototype craft, [3].

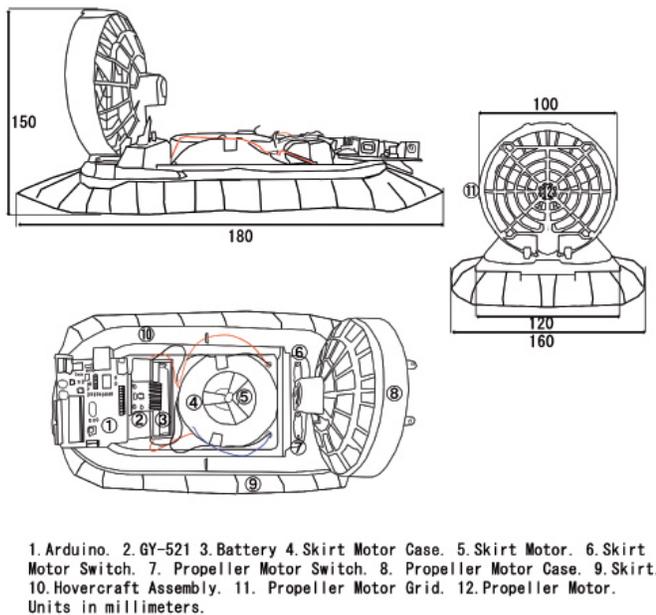


Figure 1. Unmanned Hovercraft schematic.

The hovercraft consists of fans and a cushion where air pressure inside the cushion enables it to float and move smoothly on any surface. The pressure inside the cushion needs to be maintained at all times, whereas the lift fan is able to operate for long periods and in all types of climates to ensure the hovercraft can move forward at certain speeds. Furthermore, the unmanned hovercraft has less friction due to the air pressure inside the hovercraft's cushion. This air reduces the friction between land or water surfaces that have direct contact with the hovercraft's skirt. This system can

also be launched from any place, whereas a larger vehicle cannot. Some disadvantages when using hovercrafts are that they require a lot of air and are loud due to fan or propeller rotation during their operation. In addition, the hovercraft has the potential to damage its skirt or cushion.

The main challenge when designing controls for underactuated systems is the non-linearity of the equations of motion that govern the dynamics, together with the manipulation of those equations so that a controller can be found. The application of any method is in general a rather difficult task, because getting the needed controller involves solving ordinary and partial differential equations. Generally, control strategies for the stabilization of underactuated systems can be found in the literature. Some of the previous studies conducted by several researchers on stabilizing the underactuated unmanned hovercraft system are mentioned and analyzed below.

In the hovercraft modeled by [4], the design consists of one powerful hovering motor and four horizontally mounted propulsion motors. A microcontroller acquired input data from the sensors and provided outputs signals to vary the speed of each motor and then perform the necessary stabilization. To this work, the proportional integral derivative (PID) controller was designed to control the hovercraft.

The nonholonomic autonomous underactuated underwater vehicle (AUV) modeled by [5] consists of regulating the dynamic model in the horizontal plane to a point with a desired orientation. A discontinuous, adaptive state feedback controller is derived that yields convergence of the trajectories of the closed loop system in the presence of parametric modeling uncertainty. To this work, the formulation of the Lyapunov-based, adaptive, smooth control law was applied.

In the paper by [6, 7] the vehicle was designed with two different control strategies for stabilizing the surge, sway and angular velocities with

different controllers. The authors used the surge force and the angular torque as inputs to the system. In addition, the mathematical model was derived based on Newton's Second Law and Euler-Lagrange. A Lyapunov controller formulation was used.

In [8], the author used an amphibious hovercraft, the Electro Cruiser, as his experimental model. An electric motor was used to drive both propellers and another one of the propellers to provide lift by keeping a low pressure air cavity inside the skirt. The dynamical model for the hovercraft was derived using the Newton-Euler method. The controller strategy was not tested.

In the paper done by [9], the nonlinear control was used to study an amphibious hovercraft. Here the hydrodynamic and aerodynamic coefficients with speed roll angle and sideslip angle were considered. They introduced an adaptive multiple model approach to acquire a linearized model of the hovercraft setting some work points according to ship speed with local controllers, and switch rule bases on weighting methods.

In the work presented at [10], a remote controlled hovercraft was modeled using Newton's Second Law where the hovercraft had two thrust fans and another one for lift providing two separate sources of input. An open loop and closed loop behavior of the system was simulated in Simulink. The author mentioned that the mathematical model was successfully and accurately controlled.

The previous work using Direct Lyapunov Approach (DLA) presented in [11, 12, 13] is taken as the starting point of this formulation for the design of the stabilizing nonlinear control law of underactuated hovercraft systems. The attractiveness of the DLA used in the formulation is that this method offers a wider range of applications and the obtained linear algebraic equations (LAEs), ordinary differential equations (ODEs), and partial differential equations (PDEs) are more tractable than those obtained with early methods applied for

the controller design of underactuated mechanical systems, [14].

The objective of this work was to apply the Direct Lyapunov Approach based method to control an unmanned hovercraft system. The Lyapunov stability was performed and simulated to illustrate the efficacy of the designed control law.

2. Dynamic and Modeling Analysis of the System

The dynamic equations of motion governing the behavior of the autonomous hovercraft with holonomic constraints are determined from the Euler-Lagrange equations, namely,

$$\frac{d}{dt} \left(\frac{\partial L(\mathbf{q}, \dot{\mathbf{q}})}{\partial \dot{\mathbf{q}}} \right) - \frac{\partial L(\mathbf{q}, \dot{\mathbf{q}})}{\partial \mathbf{q}} = [\mathbf{M}(\mathbf{q})]\ddot{\mathbf{q}} + [\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \boldsymbol{\tau} \quad (1)$$

where $\mathbf{q} \in \mathfrak{R}^n$ represents the vector of the generalized coordinates, with x , y and ψ representing the generalized hovercraft position and orientation in the earth fixed coordinates. $\dot{\mathbf{q}}$, and $\ddot{\mathbf{q}} \in \mathfrak{R}^n$ represent velocities and accelerations, respectively, for the $n=3$ degrees of freedom of the hovercraft system. $L(\mathbf{q}, \dot{\mathbf{q}}): \mathfrak{R}^{2n} \rightarrow \mathfrak{R}$ is the Lagrangian defined as the kinetic energy minus the potential energy of the system. The right-hand side of Eq. (1), specified as $\boldsymbol{\tau} \in \mathfrak{R}^n$, consists of the actuation for the degrees of freedom. It is assumed that the degrees of freedom are ordered so that the first m elements of the right side vector contain the nonzero inputs. For an underactuated system, only m of the inputs are nonzero where $m < n$. In the dynamic equations of motion (1), $[\mathbf{M}(\mathbf{q})] \in \mathfrak{R}^{n \times n}$ is the positive definite mass and/or inertia matrix, $[\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} \in \mathfrak{R}^n$ consists of centripetal and Coriolis forces and/or moments, and $\mathbf{G}(\mathbf{q}) \in \mathfrak{R}^n$ consists of forces and/or moments stemming from gradients of conservative fields.

The requirement of the control law is to stabilize the system and in order to achieve this,

the Lyapunov second method is applied for its development.

The control challenge arises from the nonlinear nature of the governing equations and the underactuation. The candidate Lyapunov function is made of intrinsically positive quantities, part of which is described as a quadratic matrix product, [15]. The goal of this effort is to use a trial Lyapunov function

$$V(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} \dot{\mathbf{q}}^T [\mathbf{K}_D] \dot{\mathbf{q}} + \Phi(\mathbf{q}) \quad (2)$$

where $V(\mathbf{q}, \dot{\mathbf{q}}) : \mathfrak{R}^{2n} \rightarrow \mathfrak{R}$ is the candidate Lyapunov function, $\Phi(\mathbf{q})$ is a real scalar potential function of the generalized coordinates, and where $\mathbf{K}_D \in \mathfrak{R}^{n \times n}$ is a symmetric, positive matrix defined as the product

$$\mathbf{K}_D = \mathbf{P}(\mathbf{q})\mathbf{M}(\mathbf{q}) \quad (3)$$

where $\mathbf{P}(\mathbf{q}) \in \mathfrak{R}^{n \times n}$ is a matrix defined so that \mathbf{K}_D has the previously mentioned specified properties.

The time derivative of the candidate function is made non-positive and this concept is the basis for the Lyapunov application to nonlinear control problems. The time derivative of Eq. (2), together with the equations of motion results in an equation that is solved by a matching method. When this method is applied, the quadratic terms in the velocities are grouped together obtaining a set of linear ordinary differential equations (ODEs). These equations are called the first matching condition, [15].

Grouping linear terms in the velocities results in linear algebraic equations (LAEs) and these equations are called the second matching condition.

The third matching condition involves only position coordinates resulting in linear partial differential equations (PDEs). This methodology is called the direct Lyapunov approach (DLA). The attractiveness of the DLA used in the formulation is

that this method offers a wider range of applications and the obtained LAEs, ODEs, and PDEs are more tractable than those obtained with early methods applied for the controller design of underactuated mechanical systems.

3. Hovercraft Model

Figure 2 shows the geometry of the hovercraft. Considering a local inertial system implies neglecting Coriolis forces induced by the rotation of the earth and consider the earth as a system locally flat. From the Figure 2 x, y and ψ represent the generalized position and orientation in the earth fixed coordinates.

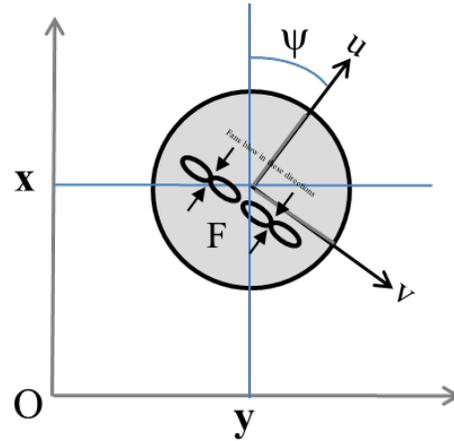


Figure 2. The simplified Hovercraft Model.

The kinematics in the inertial [16, 17, 18, 19] system that involves the hovercraft can be expressed as

$$\left. \begin{aligned} \dot{x} &= \cos(\psi)u - \sin(\psi)v \\ \dot{y} &= \sin(\psi)u + \cos(\psi)v \\ \dot{\psi} &= r \end{aligned} \right\} \quad (4)$$

Manipulating and rearranging terms from Eq. (4)

$$\begin{aligned} v &= \dot{y} \cos(\psi) - \dot{x} \sin(\psi) \\ u &= \dot{x} \cos(\psi) + \dot{y} \sin(\psi) \\ \dot{\psi} &= r \end{aligned} \quad (5)$$

where u is the surge velocity vector, v is the sway velocity vector, and r is the yaw angular velocity vector.

The Euler-Lagrange equation for this system is

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{q} = \begin{bmatrix} \mathbf{F} \\ \boldsymbol{\tau} \\ 0 \end{bmatrix}. \quad (6)$$

where \mathbf{F} denotes the control force in the surge direction and $\boldsymbol{\tau}$ denotes the control torque in yaw. The control torque is a function of \mathbf{F} and its perpendicular distance from the center of the fan to the center of mass of the hovercraft.

Note that in order to obtain a simple model capturing essential nonlinearities of the hovercraft, the inertia matrix was assumed to be diagonal and constant. If \mathbf{M} is constant, the Coriolis and centripetal matrix is equal to zero. The hydrodynamic damping was cancelled given that it is not used in controlling the system.

To use a direct Lyapunov method for designing a control law, [20, 21, 22], the time derivative of Eq. (2) is computed and it produces

$$\begin{aligned} \dot{V} &= \dot{\mathbf{q}}^T \mathbf{K}_D \ddot{\mathbf{q}} + \frac{1}{2} \dot{\mathbf{q}}^T \dot{\mathbf{K}}_D \dot{\mathbf{q}} + \dot{\mathbf{q}}^T \nabla \phi(\mathbf{q}) \\ &= \dot{\mathbf{q}}^T \mathbf{K}_D \mathbf{M}^{-1}(\mathbf{q}) \left(-\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{s} - \begin{bmatrix} \mathbf{F} \\ \boldsymbol{\tau} \\ 0 \end{bmatrix} \right) \\ &\quad + \frac{1}{2} \dot{\mathbf{q}}^T \dot{\mathbf{K}}_D \dot{\mathbf{q}} + \dot{\mathbf{q}}^T \nabla \phi(\mathbf{q}) = -\dot{\mathbf{q}}^T \mathbf{K}_v \dot{\mathbf{q}}. \end{aligned} \quad (7)$$

Following the procedures of [11], we decompose Eq. (7) into three matching equations. Since the \mathbf{K}_D is a constant matrix this leads to

$$-\mathbf{K}_D \mathbf{M}(\mathbf{q})^{-1} \mathbf{M}(\mathbf{q})^{-1} \mathbf{K}_D = 0. \quad \dot{\mathbf{K}} = 0 \quad (8)$$

The Second Matching Equation, after expressing \mathbf{F} as $F = F_1 \dot{\mathbf{q}}$ and rearranging the $\dot{\mathbf{q}}$ terms, is

$$\begin{bmatrix} \mathbf{F}_1 \\ \boldsymbol{\tau}_1 \\ 0 \end{bmatrix} = -\mathbf{P}(\mathbf{q})^{-1} \mathbf{K}_v \quad (9)$$

for which the solution is

$$\mathbf{K}_v = \sum_{i=1}^m \alpha_i \mathbf{P}_i \mathbf{P}_i^T \quad (10)$$

where the α_i are constants chosen so that \mathbf{K}_v is positive semi-definite and \mathbf{P}_i is the i^{th} column of $\mathbf{P}(\mathbf{q})$.

The control law contribution from the second matching condition is the product of \mathbf{F}_1 and $\dot{\mathbf{q}}$.

The third matching equation is stated as, where the first m equations in Eq. (6) are used to determine the control law contribution while the last $n - m$ rows of the equation provide linear, first order partial differential equations for the potential as seen in

$$-\mathbf{P}(\mathbf{q})\mathbf{G}(\mathbf{q}) + \mathbf{P}(\mathbf{q}) \begin{bmatrix} \mathbf{F}_2 \\ \boldsymbol{\tau}_2 \\ 0 \end{bmatrix} + \nabla \Phi(\mathbf{q}) = 0 \quad (11)$$

where $\mathbf{G}=0$

In taking the time derivative of the candidate Lyapunov function, the potential is assumed to be a function of the generalized positions \mathbf{q} alone. At this time it is important to mention that the potential is also needed. In order to assure the stability condition of the system, the Hessian of the potential must be positive definite. The Hessian which denotes the second derivative of the potential with respect to the generalized coordinates, is given by

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 \Phi(\mathbf{q}_1, \mathbf{q}_2)}{\partial \mathbf{q}_1 \partial \mathbf{q}_1} & \frac{\partial^2 \Phi(\mathbf{q}_1, \mathbf{q}_2)}{\partial \mathbf{q}_1 \partial \mathbf{q}_2} \\ \frac{\partial^2 \Phi(\mathbf{q}_1, \mathbf{q}_2)}{\partial \mathbf{q}_2 \partial \mathbf{q}_1} & \frac{\partial^2 \Phi(\mathbf{q}_1, \mathbf{q}_2)}{\partial \mathbf{q}_2 \partial \mathbf{q}_2} \end{bmatrix} \quad (12)$$

and the necessary condition on $|\mathbf{H}|$ is

$$\det(\mathbf{H}) > 0. \quad (13)$$

In order to guarantee that Eq. (12) is a positive definite matrix, its eigenvalues are required to all be positive. The method to solve the third matching equation is similar to the matching equations developed for stabilization as shown in [15].

The different parameters are chosen such that the eigenvalues of the linearized system are the same as those chosen for stabilization.

The Hessian of the potential is tested so that the potential is concave upward at the equilibrium point. It is a convenient way to choose the parameters. The stabilization will be achieved once all the mentioned constrains are satisfied. Lyapunov also needs to be tested. Testing the control law through simulation will verify the reliability of the process. To simulate the systems, the quantities of \mathbf{K}_D , \mathbf{K}_v , the potential, the control inputs, and the coefficients are brought from Maple to MATLAB. The control design is first done in Maple.

4. System Model Results

The control law design method is applied to the hovercraft system in order to drive the states from a given initial condition to the origin and stabilizing them at that point. Numerical simulation, done using MATLAB, confirms that the nonlinear control law stabilizes the system. The simulation results presented in the plots of Figure 3 and Figure 4 illustrate the hovercraft position and velocity as well as the orientation angle and angular velocity as a function of time, respectively.

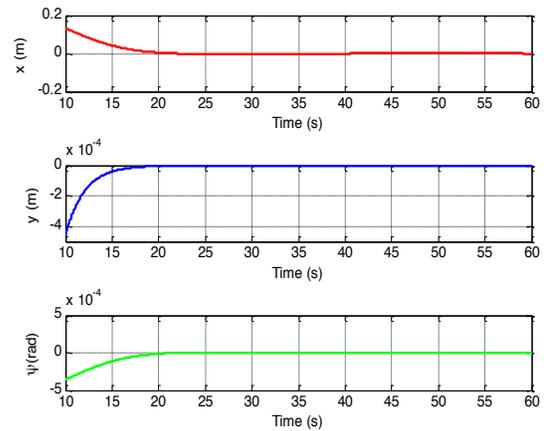


Figure 3. Stabilization of the Hovercraft (Generalized position and orientation).

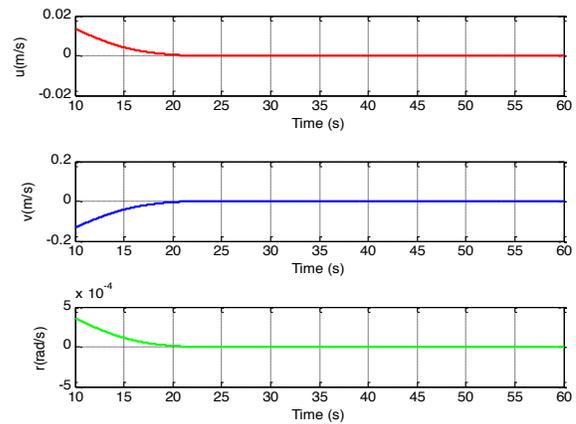


Figure 4. Velocity variables for Stabilization of the Hovercraft.

The following figures show the Lyapunov function performance and its first time derivative, as well as the control law.

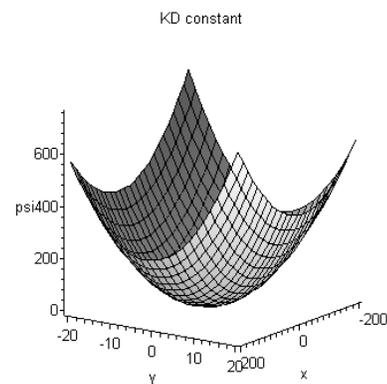


Figure 5. Hovercraft Potential (x-y plane).

Figure 6 presents a 3D plot of the potential for the interval $(-100, 100)$ for Ψ and $(-100, 100)$ for y . The proper shape of the potential is demonstrated for the hovercraft stabilization.

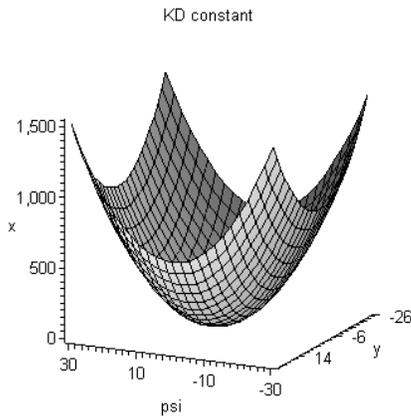


Figure 6. Hovercraft Potential (y - Ψ plane).

The following figures show the Lyapunov function performance and its first time derivative, as well as the control law.

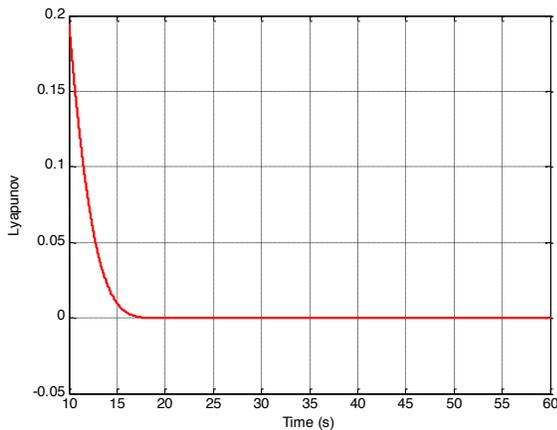


Figure 7. Lyapunov Time History.

The behavior shown in Figures 7 and 8 demonstrate the validity of the Lyapunov candidate function candidate function because it is monotonically decreasing with time for the hovercraft stabilization.

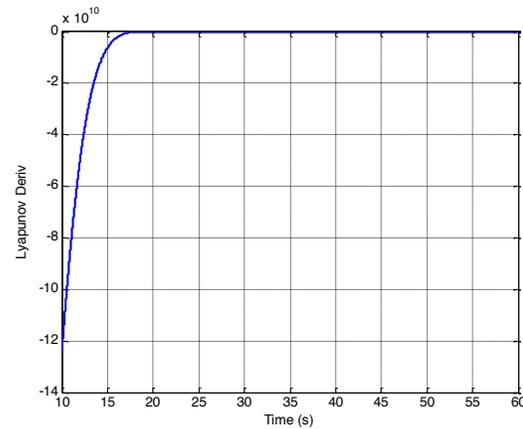


Figure 8. Lyapunov Time Derivative.

The behavior of the control law to stabilize the hovercraft system is shown in Figure 9 and Figure 10 for F and τ , respectively. Simulation results are presented to illustrate the efficacy of the designed control law.

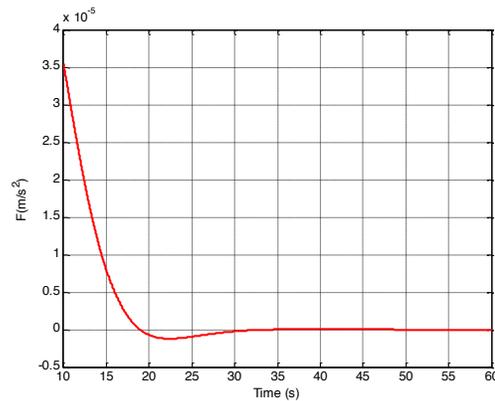


Figure 9. Control Law (F)

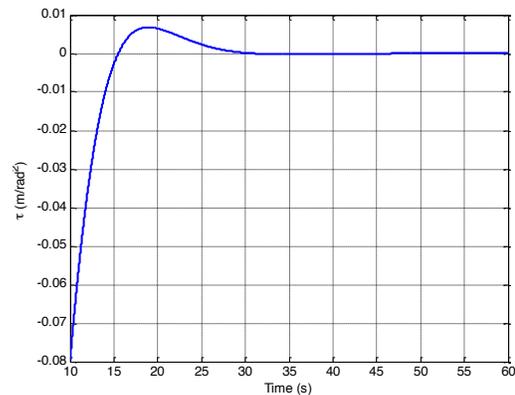


Figure 10. Control Law (τ).

5. Conclusions and Future Work

This work introduces methods as applied to a hovercraft that can be used to simulate the behavior of the underactuated system with three degrees of freedom and two control inputs.

A scheme based on a Lyapunov approach to stabilize the surge, sway, and angular velocity of yaw has been proposed to design a controller.

The simulated model was used to test the control law showing the best stability performance. Using the design control law on a prototype within a microcontroller Arduino, different disturbances affecting the stability of the system have been tested.

Modeling and control technologies are required to assure that the prototype will perform safely, reliably, and robustly in the presence of disturbances and weight rising. This part of the work is currently in progress.

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